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Dynamical equations from a first-order perturbative superspace formulation of 10D, $\mathcal{N}=1$ string-corrected supergravity, (I)*

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Abstract: Utilizing a first-order perturbative superspace approach, we derive the bosonic equations of motion for the 10D, $\mathcal{N}=1$ supergravity fields. We give the lagrangian corresponding to these equations derived from superspace geometry. Moreover, the equivalence of this lagrangian to the first-order perturbative component level lagrangian of anomaly-free supergravity is proven. Our treatment covers both the two-form and six-form formulations.

Keywords: Supersymmetry and Duality, Supergravity Models, Supersymmetric Effective Theories, Superspaces.

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1. Introduction

Some years ago [1]–[5], we began to set up a framework for development of a perturbative superspace method to describe the effect of string corrections to the equations of motion for 10D, $\mathcal{N} = 1$ supergravity. At that time, we emphasized the importance of a perturbative approach as being best suited to such an application. Today such perturbative modifications to superspace geometry are often called "deformations."

A little prior to the appearance of the work in [1], there was a *suggestion* [6] that it should be possible to use superspace Bianchi identities to find the string corrected equations of motion for 10D, $\mathcal{N} = 1$ supergravity. However, this work *contained no* concrete or clear indication of how this might be done. After the appearance of [1] Nilsson [7] performed an analysis of the usual superspace geometry, based on the constraint $T_{\alpha\beta}{}^{\underline{c}} = i\,(\sigma^{\underline{c}})_{\alpha\beta}$ (in our notation), and concluded that it requires modification in the presence of string corrections.

Other groups (see references [3]–[13] in the work of [8]) during this period began to study this issue also. Two notable features of this class of works are: (a) the approach is described as being a *non-perturbative* description (i.e. contains string corrections correctly to all orders) and (b) the condition $T_{\alpha\beta}{}^{\underline{c}} = i\,(\sigma^{\underline{c}})_{\alpha\beta}$ on the lowest dimensional torsion component is imposed to *all* orders in the string corrections.

The above recitation of the published historical record is thus completely clear in determining that discussion of the perturbative deformation technique begins with the work in [1] despite impressions otherwise that may be engendered by many patterns of citation on this topic in today's literature. One of the authors of this work (SJG) claims, even at the present point in time, that the deformations identified in [4, 5] provide a correct *first-order* description of the Lorentz Chern-Simons modifications to 10D, $\mathcal{N} = 1$ supergravity and has never accepted the argument given in [8]. The reasons for this will be given in an accompanying paper [9].

In an attempt to initiate a logical and calm debate with a minimal of controversy, in this work we investigate *solely* the proposal of [4, 5]. Namely, we concentrate only on an investigation at first order in the string tension. We undertake this not to re-ignite what became an acrimonious debate but because more recent developments suggest this is the appropriate time to re-enter this debate.

At the time we first began deliberations of how closed type-I and heterotic string theory must perturbatively modify superspace geometry, we also similarly investigated how the open type-I superstring must have a similar effect on 10D, $\mathcal{N} = 1$ super Yang-Mills theory. This effort was rewarded with the discovery of the first manifestly supersymmetrical description (see first work of [3]) of the lowest order correction from open superstring theory. Again the method used was that of a perturbative solution to the appropriate superspace Bianchi identities. The component formulation of the lowest order pure Yang-Mills part of the corrections was discovered in [10]. Very shortly after our superspace description appeared in [3], an equivalent component level derivation was carried out [11] and complete agreement was found. However, neither the result in the first work of [3] nor that in [11] was derived *directly* from a superstring argument. This same result was verified yet again more recently [12] and more importantly a recursive procedure was developed to determine the explicit form of the deformation to *all* orders. But again the method used did not rely on superstring theory.

Finally in 2002, the first derivation [13] of the lowest order open string correction to superspace deformations (along with a recursive procedure to derive all orders results) was found on the basis of superstring theory. Complete agreement was found with the result in [3]. As the reader will note this is eighteen years after our first suggestion of this lowest order superspace deformation arising from the open superstring. This gulf of time is indicative of the difficulty and subtlety of direct superstring derivations of superspace descriptions of the string low energy effective action.

The "pure spinor" method used in [13] is clearly a very powerful tool for providing superstring-based derivations of the required deformations to supergeometry. With continued development (e.g. [14]), we expect that at some point in the not too distant future it will be applied to the problem of the closed type-I or heterotic superstring low energy effective action.

In light of this, we wish to have in the record (and in advance of this hoped-for breakthrough using the pure spinor technique in covariant superstring field theory) as complete as possible a description of the proposal for the superspace supergravity deformations made in the works of [4, 5]. This is the primary reason for our offering the present work for the consideration of our readers. A pure spinor derivation of the deformation should definitively settle the controversy.

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Our conventions and definitions for 10D metrics, Pauli matrices have appeared in many of the works in [1]–[5] and we have also included an additional discussion of definitions in an appendix. A mathematica package for manipulating our 10D spinor matrix algebra is available on-line via [17].

2. First-order corrected 10D, $\mathcal{N} = 1$ superspace supergravity geometry

Let us begin by reviewing the results in [5]. There a solution was given to the 10D, $\mathcal{N} = 1$ supergravity Bianchi identities that is correct to first order in the perturbative parameters β' and γ'. Here we recall the Yang-Mills truncated version (eliminating the Yang-Mills fields or equivalently putting $\beta' = 0$) of this solution and we complete the results by the presentation of the bosonic equations of motion. Also, we would like to emphasize here that this solution rests on two assumptions, both valid at first order in γ'. The first assumption is a constraint on the 0 dimensional torsion component $T_{\delta\gamma}{}^{\underline{a}}$ and the second is a choice for the usual 1 dimensional auxiliary field denoted by $A_{\underline{a}\,\underline{b}\,\underline{c}}$. Let us now discuss these two inputs and their consequences in order.

Before making any assumptions on torsion components it is always worthwhile to study purely conventional constraints, which do reduce the number of independent torsion and curvature components, but do not have any consequence on the dynamics. Using standard methods (see the work in [20]) one can show that the following set of constraints is purely conventional:

$$i\,(\sigma_{\underline{a}})^{\alpha\beta}\,T_{\alpha\beta}{}^{\underline{b}} = 16\,\delta_{\underline{a}}{}^{\underline{b}}\,, \qquad i\,(\sigma_{\underline{c}})^{\alpha\beta}\,T_{\alpha\,\underline{b}}{}^{\underline{c}} = 0\,, \tag{2.1}$$

$$i\,(\sigma_{\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}\,\underline{e}})^{\alpha\beta}\,T_{\alpha\beta}{}^{\underline{e}} = 0\,, \qquad T_{\alpha\,[\underline{d}\,\underline{e}]} = 0\,, \tag{2.2}$$

$$T_{\underline{d}\,\underline{e}\,\underline{b}} = \frac{1}{8}\,(\sigma_{\underline{d}\,\underline{e}})_{\alpha}{}^{\beta}\,T_{\beta\,\underline{b}}{}^{\alpha} + i\,\frac{1}{16}\,(\sigma_{\underline{b}})^{\alpha\beta}\,\mathcal{R}_{\alpha\,\beta\,\underline{d}\,\underline{e}}\,. \tag{2.3}$$

The role of each of these respective constraints is easy to understand. The first equation removes $\mathrm{E}_{\underline{a}}{}^{\underline{m}}$ as an independent variable. The second equation removes $\mathrm{E}_{\underline{a}}{}^{\mu}$ as an independent variable. The third constraint is a coset conventional constraint that removes part of $\mathrm{E}_{\alpha}{}^{\mu}$ as an independent variable. The fourth constraint removes $\omega_{\alpha\,\underline{b}\,\underline{c}}$ as an independent variable and the final constraint removes $\omega_{\underline{a}\,\underline{b}\,\underline{c}}$ as an independent variable. It is a simple matter to show that the torsion and curvature super tensors in [5], satisfy these conditions. Since these are purely conventional constraints, they may be imposed to *all* orders in the string slope-parameter expansion.

In addition, notice that the first and third equations of (2.3) imply that the most general structure of the zero dimensional torsion is

$$T_{\delta\gamma}{}^{\underline{a}} = i\,(\sigma^{\underline{a}})_{\delta\gamma} + i\,\frac{1}{5!}\,\left(\sigma^{[5]}\right)_{\delta\gamma}\,X_{[5]}{}^{\underline{a}}\,, \tag{2.4}$$

with $X_{[5]}{}^{\underline{a}}$ in the appropriate 1050 dimensional irrep of SO(1,9), while in [5] the following zero dimensional torsion constraint was *assumed*

$$T_{\delta\gamma}{}^{\underline{a}} = i\,(\sigma^{\underline{a}})_{\delta\gamma} + \mathcal{O}(\,(\gamma')^2\,)\,. \tag{2.5}$$

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Recall also, that Nilsson advocated already in [7] that the assumption $X_{[5]}{}^{\underline{a}} = 0$ is incompatible with the inclusion of higher than second order curvature terms in the effective action. In this logic the vanishing of this 0 dimensional superfield can be valid only at first order in γ' — the regime in which [5] was written. It is worthwhile to note the possibility that $X_{[5]}{}^{\underline{a}}$ contributes to higher order terms,

$$X_{[5]}{}^{\underline{a}} = \mathcal{O}((\gamma')^2)\,. \tag{2.6}$$

But as the work in [5] was only to first order, it was completely moot on this point.

Now let us begin to write the solution of the Bianchi identities for the torsion subject to the conventional constraints (2.3) and assumption (2.5):

$$T_{\alpha\beta}{}^{\gamma} = -\left[\,\delta^{\gamma}_{(\alpha}\delta^{\delta}_{\beta)} + (\sigma^{\underline{a}})_{\alpha\beta}(\sigma_{\underline{a}})^{\gamma\delta}\,\right]\chi_\delta\,, \tag{2.7}$$

$$T_{\alpha\underline{b}}{}^{\gamma} = \frac{1}{48}(\sigma_{\underline{b}}\sigma^{[3]})_\alpha{}^\gamma A_{[3]}\,, \tag{2.8}$$

$$T_{\underline{a}\,\underline{b}\,\underline{c}} = -2\,L_{\underline{a}\,\underline{b}\,\underline{c}}\,, \tag{2.9}$$

$$\mathcal{R}_{\alpha\,\beta\;\underline{a}\,\underline{b}} = i\,2(\sigma^{\underline{c}})_{\alpha\beta}(\,L_{\underline{a}\,\underline{b}\,\underline{c}} - \frac{1}{8}A_{\underline{a}\,\underline{b}\,\underline{c}}\,)\; - \;i\,\frac{1}{24}(\sigma_{\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}\,\underline{e}})_{\alpha\beta}A^{\underline{c}\,\underline{d}\,\underline{e}}\,, \tag{2.10}$$

$$\nabla_\alpha\chi_\beta = -i\,(\sigma^{\underline{a}})_{\alpha\beta}\nabla_{\underline{a}}\Phi + i\,\frac{1}{48}(\sigma^{[3]})_{\alpha\beta}\left(4L_{[3]} + A_{[3]} - i\frac{1}{2}(\chi\sigma_{[3]}\chi)\right), \tag{2.11}$$

$$\mathcal{R}_{\alpha\underline{c}\,\underline{a}\,\underline{b}} = i\,(\sigma_{[\underline{a}|})_{\alpha\beta}T_{\underline{c}|\,\underline{b}]}{}^{\beta} + i\gamma'\,(\sigma_{[\underline{c}|})_{\alpha\beta}\mathcal{R}^{\underline{k}\,\underline{l}}{}_{|\underline{a}\,\underline{b}]}T_{\underline{k}\,\underline{l}}{}^{\beta}\,, \tag{2.12}$$

with Φ a scalar superfield (dilaton) transforming into χ_α (dilatino) under supersymmetry,

$$\chi_\alpha = -2\,\nabla_\alpha\Phi\,, \tag{2.13}$$

and $A_{\underline{a}\,\underline{b}\,\underline{c}}$ an auxiliary superfield. Once again, we emphasize these are required *solely* to first order in γ'.

The fixing of this auxiliary field is the second input which characterizes the solution proposed in [5]. Making the *choice*

$$A_{\underline{a}\,\underline{b}\,\underline{c}} \doteq -i\,\gamma'\,(T_{\underline{k}\,\underline{l}}\,\sigma_{\underline{a}\,\underline{b}\,\underline{c}}\,T^{\underline{k}\,\underline{l}})\,, \tag{2.14}$$

the theory is put completely on shell. This means that all torsion and curvature components, as well as the spinorial derivatives of all objects in the geometry can be expressed in function of the dilaton Φ, the dilatino χ_α, the gravitino Weyl tensor sitting in its field strengths $T_{\underline{a}\,\underline{b}}{}^{\gamma}$, the Weyl tensor sitting in the curvature $\mathcal{R}_{\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}}$ together with the supercovariant object $L_{\underline{a}\,\underline{b}\,\underline{c}}$ appearing in the spacetime torsion.

The form of $A_{\underline{a}\,\underline{b}\,\underline{c}}$ written in terms of $T_{\underline{a}\underline{b}}{}^{\gamma}$ (the supercovariantized gravitino "curl") can be understood on the basis of a remarkable property: open-string/closed-string duality. This conjectured property of the low energy effective action of the superstring was made even before this property had a name. Bergshoeff and Rakowski [22] noted that in 6D simple superspace the quantities

$$T^{\underline{c}\,\underline{d}\,\gamma}\,, \qquad \mathcal{R}_{\underline{a}\,\underline{b}}{}^{\underline{c}\,\underline{d}} \tag{2.15}$$

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share many common properties with the fields of a vector multiplet

$$\lambda^{\gamma\,\widehat{I}}\,, \qquad F_{\underline{a}\,\underline{b}}{}^{\widehat{I}} \tag{2.16}$$

and thus asserted that large numbers of higher derivative supergravity terms may be treated as if one were coupling a vector multiplet to the supergavity multiplet. The result in [5] implements this strategy for the 10D, $\mathcal{N} = 1$ superspace and after this work Bergshoeff and de Roo [23] extended this approach to 10D theories at the level of component fields.

The object $L_{\underline{a}\,\underline{b}\,\underline{c}}$ was introduced[1] for the ten dimensional theory [5] in order to permit the simple passage between the 2-form and 6-form formulation of the 10D, $\mathcal{N} = 1$ supergravity theory. It is not an independent variable but its explicit form as a function of the component fields is determined only by specifying which of the two (2-form vs. 6-form) gauge fields is in the supergravity multiplet. This will be discussed in subsequent chapters.

In particular, $L_{\underline{a}\,\underline{b}\,\underline{c}}$ must satisfy the following conditions

$$\nabla_\alpha L_{\underline{a}\,\underline{b}\,\underline{c}} = i\,\frac{1}{4}(\sigma_{[\underline{a}})_{\alpha\beta}\left(\, T_{\underline{b}\,\underline{c}]}{}^{\beta} - \gamma' \mathcal{R}^{\underline{k}\,\underline{l}}{}_{\underline{b}\,\underline{c}]}\, T_{\underline{kl}}{}^{\beta}\right), \tag{2.17}$$

$$\begin{aligned}\nabla_\alpha T_{\underline{a}\,\underline{b}}{}^{\beta} &= \frac{1}{4}(\sigma^{\underline{c}\,\underline{d}})_\alpha{}^\beta\, \mathcal{R}_{\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}} - T_{\underline{a}\,\underline{b}}{}^{\gamma}\, T_{\gamma\alpha}{}^{\beta} + \\ &\quad + \frac{1}{48}\Big[\, 2L_{\underline{a}\,\underline{b}\,\underline{c}}(\sigma^{\underline{c}}\sigma^{[3]})_\alpha{}^\beta - (\sigma_{[\underline{a}|}\sigma^{[3]})_\alpha{}^\beta \nabla_{|\underline{b}]}\,\Big]\, A_{[3]}\,,\end{aligned} \tag{2.18}$$

in order for the Bianchi identities on the superspace torsions and curvatures to be satisfied. These same Bianchi identities require

$$\nabla_{\underline{a}}\chi_\beta = -i\,\frac{1}{2}(\sigma^{\underline{b}})_{\alpha\beta}(T_{\underline{a}\,\underline{b}}{}^{\alpha} - 2\gamma'\mathcal{R}^{\underline{k}\,\underline{l}}{}_{\underline{ab}}\, T_{\underline{k}\,\underline{l}}{}^{\alpha})\,, \tag{2.19}$$

$$\begin{aligned}(\sigma^{\underline{a}\,\underline{b}})_\beta{}^\alpha T_{\underline{a}\,\underline{b}}{}^{\beta} &= -i\,8\,(\sigma^{\underline{a}})^{\alpha\beta}\chi_\beta\nabla_{\underline{a}}\Phi - i\frac{1}{24}(\sigma^{[3]})^{\alpha\beta}\chi_\beta\,(\,16L_{[3]} + A_{[3]}\,) + \\ &\quad + 3\gamma'(\sigma^{\underline{a}\,\underline{b}})_\beta{}^\alpha \mathcal{R}^{\underline{k}\,\underline{l}}{}_{\underline{a}\,\underline{b}}\, T_{\underline{k}\,\underline{l}}{}^{\beta} \quad .\end{aligned} \tag{2.20}$$

The results given above are sufficient to derive the equations of motion for the spinors, already presented in [5], and we will now use them in order to derive the bosonic equations of motion. A detailed presentation of using superspace techniques for deriving equations of motion can be found in [15, 16]

In order to find the equation of motion of the scalar let us begin with the relation (2.19) multiplied by a sigma matrix $(\sigma^a)^{\gamma\alpha}$ and differentiate it with ∇_β,

$$\begin{aligned}(\sigma^{\underline{a}})^{\gamma\alpha}\nabla_{\underline{a}}\,(\nabla_\beta\chi_\alpha) &= i\,\frac{1}{2}(\sigma^{\underline{a}\,\underline{b}})_\alpha{}^\gamma\nabla_\beta\left(T_{\underline{a}\,\underline{b}}{}^{\alpha} \;-\; 2\gamma'\mathcal{R}^{\underline{k}\,\underline{l}}{}_{\underline{a}\,\underline{b}}\, T_{kl}{}^{\alpha}\right) + \\ &\quad + (\sigma^{\underline{a}})^{\gamma\alpha}\left[\nabla_{\underline{a}}, \nabla_\beta\right]\chi_\alpha\,.\end{aligned} \tag{2.21}$$

Notice that the l.h.s. contains the spacetime derivatives of both $(\sigma^{\underline{b}})_{\beta\alpha}\nabla_{\underline{b}}\Phi$ and $(\sigma^{[3]})_{\beta\alpha}$ $L_{[3]}$, while the r.h.s. can be computed using at most three-half dimensional results recalled above. Therefore, one obtains the equation of motion of the scalar from (2.21) by taking

[1]The first appearance of the L-type variable in the physics literature occurred in the work of [18]. It was introduced to permit a unified superspace description of theories related one to another by Poincaré duality.

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the trace ${\delta_\gamma}^\beta$

$$16\nabla^{\underline{a}}\nabla_{\underline{a}}\Phi = 4\mathcal{R} \; - \; 8\gamma'\mathcal{R}^{\underline{k}\,\underline{l}\,\underline{a}\,\underline{b}}\mathcal{R}_{\underline{k}\,\underline{l}\,\underline{a}\,\underline{b}} + \text{fermions}\,. \tag{2.22}$$

Moreover, the same relation (2.21), if multiplied by $(\sigma_{\underline{e}\,\underline{f}})_\gamma{}^\beta$, yields

$$\nabla^{\underline{a}}L_{\underline{a}\,\underline{e}\,\underline{f}} = -4L_{\underline{a}\,\underline{e}\,\underline{f}}\nabla^{\underline{a}}\Phi + \text{fermions}\,. \tag{2.23}$$

The remaining independent part of (2.21) can be projected out if one multiplies it by $(\sigma_{efgh})_\gamma{}^\beta$. The obtained relation together with the Bianchi identity for the torsion with only vectorial indices gives

$$\nabla_{[\underline{e}}L_{\underline{f}\,\underline{g}\,\underline{h}]} = -3L_{[\underline{e}\,\underline{f}}{}^{\underline{a}}L_{\underline{g}\,\underline{h}]\,\underline{a}} - \frac{3}{2}\gamma'\mathcal{R}_{\underline{k}\,\underline{l}\,[\underline{e}\,\underline{f}}\mathcal{R}^{\underline{k}\,\underline{l}}{}_{\underline{g}\,\underline{h}]} + \text{fermions}\,. \tag{2.24}$$

Notice that (2.23) and (2.24) suggest that the object $L_{\underline{abc}}$ might be *either* related to the field strengths of a two-form *or* dual field strength of a six-form depending on which of these two equations is interpreted as the Bianchi identity and which is as the equation of motion.

For example, assuming that (2.23) gives the equation of motion for a two-form gauge field, then (2.24) must correspond to its Bianchi identity. Searching for a closed three-form in the geometry, in which the field strengths of this two-form can be identified, one might want to use the identity satisfied by a Lorentz Chern-Simons three-form Q[2]

$$\nabla_{[\underline{e}}Q_{\underline{f}\,\underline{g}\,\underline{h}]} - \frac{3}{2}T_{[\underline{e}\,\underline{f}}{}^{\underline{a}}Q_{\underline{g}\,\underline{h}]\,\underline{a}} = -\frac{3}{2}\mathcal{R}_{[\underline{e}\,\underline{f}|\underline{k}\,\underline{l}}\,\mathcal{R}_{|\underline{g}\,\underline{h}]}{}^{\underline{k}\,\underline{l}} + \text{fermions} \tag{2.25}$$

in order to "absorb" the curvature squared term in the r.h.s. of (2.24) .

Observe that the structure of the equations (2.24) and (2.25) is almost the same, with the only difference that in the r.h.s. the role of the "group" indices and "form" indices of the curvature are exchanged with respect to one another. Since the curvature is defined by a connection with torsion, it is not symmetric with respect to the exchange of its pairs of indices. Therefore, $(L - \gamma'Q)_{\underline{abc}}$ cannot be equal exactly to the vectorial component of a closed three-form, but their difference is an object which serves as a link between the two curvature squared expressions we have in (2.24) and (2.25). This object (called "$Y_{\underline{abc}}$" in the next chapter) does exist as was first demonstrated in [5]. After it has been properly identified, we can use $Y_{\underline{abc}}$ to show

$$\nabla_{[\underline{e}}(L - \gamma'Q - \gamma'Y)_{\underline{f}\,\underline{g}\,\underline{h}]} - \frac{3}{2}T_{[\underline{e}\,\underline{f}}{}^{\underline{a}}(L - \gamma'Q - \gamma'Y)_{\underline{g}\,\underline{h}]\,\underline{a}} = \text{fermions} \tag{2.26}$$

at first order in γ'. This is the relation, which shows that (at least modulo fermionic contributions) $(L - \gamma'Q - \gamma'Y)_{\underline{abc}}$ can be identified as the vectorial component of a closed three-form.

Conversely assuming that (2.24) gives the equation of motion for a two-form gauge field, then (2.23) must correspond to its Bianchi identity in the dual theory. This theory is slightly easier to construct because although it contains the first order corrections superstring corrections, it does not require a dual Chern-Simons term for its consistency.

[2]The second two indices on the Riemann curvature tensor may be thought of as the Lie algebraic "group" indices for SO(1, 9).

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Finally, the Ricci tensor and the scalar curvature can be derived from (2.20) using the dimension three-half results

$$\frac{1}{2}\mathcal{R}^{(\underline{d}\,\underline{c})} = 2\nabla^{(\underline{d}}\nabla^{\underline{c})}\Phi + 2\,\gamma'\,\mathcal{R}^{\underline{k}\,\underline{l}\,\underline{d}\,\underline{b}}\,\mathcal{R}_{\underline{k}\,\underline{l}}{}^{\underline{c}}{}_{\underline{b}} + \text{fermions}\,, \tag{2.27}$$

$$\mathcal{R} = -16\nabla^{\underline{a}}\Phi\nabla_{\underline{a}}\Phi + \frac{2}{3}L^{\underline{a}\,\underline{b}\,\underline{c}}L_{\underline{a}\,\underline{b}\,\underline{c}} + 3\gamma'\,\mathcal{R}^{\underline{k}\,\underline{l}\,\underline{a}\,\underline{b}}\,\mathcal{R}_{\underline{k}\,\underline{l}\,\underline{a}\,\underline{b}} + \text{fermions}\,. \tag{2.28}$$

Throughout our discussion up to this point, we were working directly with the superfields of 10D, $\mathcal{N} = 1$ superspace supergravity. So all equations were superspace equations. For the rest of this paper, we will set *all* fermions to zero. We will utilize the same symbols to denote the various quantities however. We establish the following notation for the purely bosonic equations found from the superspace Bianchi identities,

$$\hat{\mathcal{E}}_{\Phi} \doteq 4\nabla^{\underline{a}}\nabla_{\underline{a}}\Phi \;-\; \mathcal{R} + 2\gamma'\,\mathcal{R}^{\underline{k}\,\underline{l}\,\underline{a}\,\underline{b}}\,\mathcal{R}_{\underline{k}\,\underline{l}\,\underline{a}\,\underline{b}}\,, \tag{2.29}$$

$$\hat{\mathcal{E}}_{B_{\underline{e}\,\underline{f}}} \doteq \nabla_{\underline{a}}\left(e^{4\Phi}L^{\underline{a}\,\underline{e}\,\underline{f}}\right), \tag{2.30}$$

$$\hat{\mathcal{E}}_{\widetilde{B}^{\underline{e}\,\underline{f}\,\underline{g}\,\underline{h}}} \doteq \nabla_{[\underline{e}}(\,L - \gamma' Q - \gamma' Y)_{\underline{f}\,\underline{g}\,\underline{h}]} - \frac{3}{2}T_{[\underline{e}\,\underline{f}}{}^{a}(L - \gamma' Q - \gamma' Y)_{\underline{g}\,\underline{h}]\underline{a}}\,, \tag{2.31}$$

$$\hat{\mathcal{E}}_{\eta_{\underline{d}\underline{c}}} \doteq \frac{1}{2}\mathcal{R}^{(\underline{d}\,\underline{c})} \;-\; 2\nabla^{(\underline{d}}\nabla^{\underline{c})}\Phi \;-\; 2\gamma'\mathcal{R}^{\underline{k}\,\underline{l}\,\underline{d}\underline{b}}\,\mathcal{R}_{\underline{k}\,\underline{l}}{}^{\underline{c}}{}_{\underline{b}}\,, \tag{2.32}$$

$$\hat{\mathcal{E}}_{\eta} \doteq \mathcal{R} + 16\nabla^{\underline{a}}\Phi\nabla_{\underline{a}}\Phi - \frac{2}{3}\,L^{\underline{a}\,\underline{b}\,\underline{c}}L_{\underline{a}\,\underline{b}\,\underline{c}} - 3\gamma'\mathcal{R}^{\underline{k}\,\underline{l}\,\underline{a}\,\underline{b}}\,\mathcal{R}_{\underline{k}\,\underline{l}\,\underline{a}\,\underline{b}}\,. \tag{2.33}$$

In order for the superspace Bianchi identities to be satisfied all of the $\widehat{\mathcal{E}}$-quantities are required to vanish. The question with which we shall wrestle for the rest of this paper is, "Does there exist a component level action whose variations lead to equations of motion that are compatible with (2.29)–(2.33)?" This same action must also contain a field such that either (2.30) or (2.31) can be interpreted as a Bianchi identity.

3. Bosonic terms of a component action for two-form formulation

The non-vanishing components of the modified 3-form field strength to this order can be written as (below we have used a slightly different set of conventions from [5] as discussed in an appendix)

$$H_{\alpha\beta\,\underline{c}} = i\frac{1}{2}(\sigma_{\underline{c}})_{\alpha\beta} + i\,4\,\gamma'(\sigma_{\underline{a}})_{\alpha\beta}\,G^{\underline{a}\,\underline{e}\,\underline{f}}G_{\underline{c}\,\underline{e}\,\underline{f}}\,, \tag{3.1}$$

$$H_{\alpha\underline{b}\,\underline{c}} = i\,2\,\gamma'\left[\,(\sigma_{[\underline{b}|})_{\alpha\beta}T_{\underline{e}\underline{f}}{}^{\beta} - 2(\sigma_{[\underline{e}]})_{\alpha\beta}T_{\underline{f}\,|\underline{b}|}{}^{\beta}\,\right]G_{\underline{c}]}{}^{\underline{e}\,\underline{f}}\,, \tag{3.2}$$

$$H_{\underline{a}\,\underline{b}\,\underline{c}} = G_{\underline{a}\,\underline{b}\,\underline{c}} + \gamma' Q_{\underline{a}\,\underline{b}\,\underline{c}}\,. \tag{3.3}$$

In the limit where $\gamma' = 0$ these equations correspond to the superspace geometry in a string-frame description of the pure supergravity theory. As was pointed out some time ago [19], the field *independence* of the leading term in the $G_{\alpha\beta\underline{c}}$ component of the 3-form field strength is indicative of this.

The quantity $L_{\underline{a}\,\underline{b}\,\underline{c}}$ in this formulation is defined by,

$$L_{\underline{a}\,\underline{b}\,\underline{c}} \doteq G_{\underline{a}\,\underline{b}\,\underline{c}} + \gamma' Q_{\underline{a}\,\underline{b}\,\underline{c}} + \gamma'\,Y_{\underline{a}\,\underline{b}\,\underline{c}} + \mathcal{O}((\gamma')^2)\,, \tag{3.4}$$

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where $G_{\underline{a}\,\underline{b}\,\underline{c}}$ is the supercovariantized field strength of a two-form, $Q_{\underline{a}\,\underline{b}\,\underline{c}}$ is the Lorentz Chern-Simons form and

$$Y_{\underline{a}\,\underline{b}\,\underline{c}} \doteq - \left(\mathcal{R}^{\underline{e}\,\underline{k}}{}_{[\underline{a}\,\underline{b}|} + \mathcal{R}_{[\underline{a}\,\underline{b}|}{}^{\underline{e}\,\underline{k}} + \frac{8}{3}\, G_{\underline{d}}{}^{\underline{e}}{}_{[\underline{a}|} G^{\underline{d}\,\underline{k}}{}_{|\underline{b}|} \right) G_{|\underline{c}]\,\underline{e}\,\underline{k}} \,. \tag{3.5}$$

This quantity, (which to our knowledge first appeared in [5]) has a remarkable property. It is a straightforward calculation to show

$$\nabla_{[\underline{e}} Y_{\underline{f}\,\underline{g}\,\underline{h}]} - \frac{3}{2} T_{[\underline{e}\,\underline{f}}{}^{\underline{a}} Y_{\underline{g}\,\underline{h}]\,\underline{a}} = - \frac{3}{2} \left(\mathcal{R}_{\underline{k}\,\underline{l}[\underline{e}\,\underline{f}} \mathcal{R}^{\underline{k}\,\underline{l}}{}_{\underline{g}\,\underline{h}]} - \mathcal{R}_{[\underline{e}\,\underline{f}|\underline{k}\,\underline{l}} \mathcal{R}_{|\underline{g}\,\underline{h}]}{}^{\underline{k}\,\underline{l}} \right) + \mathcal{O}(\gamma') \,. \tag{3.6}$$

By keeping terms *only* up to first order in γ' we find that a lagrangian density of the form

$$\mathcal{L} = \mathrm{e}^{-1} e^{4\Phi} \left[\mathcal{R}(\omega) + 16\, (\mathrm{e}^{\underline{a}}\Phi)\, (\mathrm{e}_{\underline{a}}\Phi) - \frac{1}{3}\, L^{\underline{a}\,\underline{b}\,\underline{c}} L_{\underline{a}\,\underline{b}\,\underline{c}} + \gamma' \,\mathrm{tr}(\mathcal{R}^{\underline{a}\,\underline{b}} \mathcal{R}_{\underline{a}\,\underline{b}}) \right] ,\,, \tag{3.7}$$

where ω is the torsion-less spin connection, is compatible with the set of equations of motion (2.27), (2.28), (2.30), (2.31) and Bianchi identity (2.29). If we expand the penultimate term to first order in γ' we find

$$\begin{aligned} \mathcal{L} \,=\, \mathrm{e}^{-1} e^{4\Phi} \Big[& \mathcal{R}(\omega) + 16\, (\mathrm{e}^{\underline{a}}\Phi)\, (\mathrm{e}_{\underline{a}}\Phi) - \frac{1}{3}\, G^{\underline{a}\,\underline{b}\,\underline{c}} \, \left(G_{\underline{a}\,\underline{b}\,\underline{c}} + 2\gamma'\, Q_{\underline{a}\,\underline{b}\,\underline{c}} \right) - \\ & - \frac{2}{3}\, \gamma'\, G^{\underline{a}\,\underline{b}\,\underline{c}} Y_{\underline{a}\,\underline{b}\,\underline{c}} + \gamma' \,\mathrm{tr}(\mathcal{R}^{\underline{a}\,\underline{b}} \mathcal{R}_{\underline{a}\,\underline{b}}) \Big] \,. \end{aligned} \tag{3.8}$$

It is easily seen that the action to first order in γ' when written using the Y variable takes a simple and elegant form.

Variation of this lagrangian with respect to the dilaton gives

$$\delta_\Phi \mathcal{L} \;\sim\; -4 \mathrm{e}^{-1} e^{4\Phi} \; [\, \mathcal{E}_g + 2\mathcal{E}_\Phi] \; \delta\Phi \,. \tag{3.9}$$

where $\mathcal{E}_g$ and $\mathcal{E}_\Phi$ are given by (2.33) and (2.29).

The variation with respect to the antisymmetric tensor at first seems very complicated due to the fact that its field strength appears in the Lorentz connection. However, one can write it simply as

$$\delta_B \mathcal{L} = \mathrm{e}^{-1} e^{4\Phi} \left(-\frac{2}{3} L^{\underline{a}\,\underline{b}\,\underline{c}} \,\delta L_{\underline{a}\,\underline{b}\,\underline{c}} + \gamma' \delta \,\mathrm{tr}\, (\mathcal{R}^{\underline{a}\,\underline{b}}\, \mathcal{R}_{\underline{a}\,\underline{b}}) \right) . \tag{3.10}$$

Replacing now (3.4) into the first term, we obtain the form

$$\begin{aligned} \delta_B \mathcal{L} \;\sim\; & 2\mathcal{E}_{B_{\underline{a}\underline{b}}} \delta B_{\underline{a}\,\underline{b}} - \frac{2}{3} \mathrm{e}^{-1} \gamma'\, e^{4\Phi} L^{\underline{a}\,\underline{b}\,\underline{c}} \delta_L \, (Q + Y)_{\underline{a}\,\underline{b}\,\underline{c}} + \\ & + \mathrm{e}^{-1}\, e^{4\Phi} \gamma' \delta_L \,\mathrm{tr} \left(\mathcal{R}^{\underline{a}\,\underline{b}}\, \mathcal{R}_{\underline{a}\,\underline{b}} \right) . \end{aligned} \tag{3.11}$$

The last terms in fact form a combination of variations which can be expressed in terms of zero order equations of motion for arbitrary variations of the entire object $L_{\underline{a}\,\underline{b}\,\underline{c}}$. This is shown in appendix B, where this combination is denoted symbolically by $f(\mathcal{E})$. Therefore, the variation of the lagrangian with respect to the antisymmetric tensor is

$$\delta_B \mathcal{L} \;\sim\; 2\mathcal{E}_{B_{\underline{a}\underline{b}}} \delta B_{\underline{a}\,\underline{b}} + \gamma'\, f(\mathcal{E}) \tag{3.12}$$

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with

$$f(\mathcal{E}) \sim 4\mathcal{E}_{B_{\underline{kl}}}\Phi_{\underline{k}}{}^{\underline{a}\,\underline{b}}\delta L_{\underline{l}\,\underline{a}\,\underline{b}} + \\ + 8\left[e^{4\Phi}\nabla^{\underline{a}}\left(e^{-4\Phi}\hat{\mathcal{E}}_{B_{\underline{bc}}}\right) + \left(e^{4\Phi}\hat{\mathcal{E}}_{\eta_{\underline{ak}}} - \hat{\mathcal{E}}_{B_{\underline{ak}}}\right) L_{\underline{k}}{}^{\underline{b}\,\underline{c}}\right]\delta L_{\underline{a}\,\underline{b}\,\underline{c}} - \\ - \frac{2}{3}\frac{1}{4!}\, e^{4\Phi}\,\mathcal{E}_{\widetilde{B}\underline{abcd}}\delta_L\left(\mathcal{E}_{\widetilde{B}_{\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}}}\right) + \mathcal{O}(\gamma')\,. \tag{3.13}$$

4. Bosonic terms of a component action for six-form formulation

Retaining the same the current $A_{\underline{abc}}$ specified by (2.12) we can introduce a seven-form N [21] satisfying an appropriate Bianchi identity. At the component level similar considerations have been carried out for the six-form formulation [24]. One of the remarkable things about this formulation is that in order to describe lowest order perturbative contributions to the effective does *not* require a Chern-Simons like modification to the seven-form field strength.

$$N_{\alpha\beta[5]} = i\,\frac{1}{2}\, e^{4\Phi}\,(\sigma_{[5]})_{\alpha\beta}\,, \tag{4.1}$$

$$N_{\alpha[6]} = -\frac{1}{4!}\epsilon_{[6][4]}\, e^{4\Phi}\,(\sigma^{[4]})_{\alpha}{}^{\beta}\chi_{\beta}\,, \tag{4.2}$$

$$N_{[7]} = \frac{1}{3!}\, e^{4\Phi}\left(L^{[3]} - \frac{13i}{8}\chi\sigma^{[3]}\chi\right)\epsilon_{[3][7]}\,. \tag{4.3}$$

In particular, it is the equation (2.29) which insures that the purely vectorial component of the N Bianchi identity is satisfied. Equations (2.29), (2.30), (2.32) and (2.33) contain the bosonic equations of motion for the component fields of the dual theory. Notice that in this case (2.29) identifies $L_{\underline{abc}}$ as the following function of the component fields of the dual theory

$$L_{\underline{a}\,\underline{b}\,\underline{c}} = -\frac{1}{7!}\,\epsilon_{\underline{a}\,\underline{b}\,\underline{c}[7]}\, e^{-4\Phi}\, N^{[7]}\,, \tag{4.4}$$

upon setting the fermions to zero. In the following we show that the lagrangian density

$$\mathcal{L}_d = \mathrm{e}^{-1}\, e^{4\Phi}\Bigg[\mathcal{R}(\omega) + 16\,(\mathrm{e}^{\underline{a}}\Phi)\,(\mathrm{e}_{\underline{a}}\Phi) + \frac{1}{3}(\,L - \gamma'(Q+Y))^2_{\underline{a}\,\underline{b}\,\underline{c}} + \\ + \gamma'\ \mathrm{tr}(\mathcal{R}^{\underline{a}\,\underline{b}}\mathcal{R}_{\underline{a}\,\underline{b}})\Bigg]\,, \tag{4.5}$$

is compatible with the set of equations of motion and Bianchi identity. Since our results are only valid to first order in γ' it follows that (4.5) should be more properly written as

$$\mathcal{L}_d = \mathrm{e}^{-1}\, e^{4\Phi}\Bigg[\mathcal{R}(\omega) + 16\,(\mathrm{e}^{\underline{a}}\Phi)\,(\mathrm{e}_{\underline{a}}\Phi) + \frac{1}{3}\,L^{\underline{a}\,\underline{b}\,\underline{c}}\,L_{\underline{a}\,\underline{b}\,\underline{c}} - \\ - \frac{2}{3}\gamma'\,L^{\underline{a}\,\underline{b}\,\underline{c}}\,Q_{\underline{a}\,\underline{b}\,\underline{c}} - \frac{2}{3}\gamma'\,L^{\underline{a}\,\underline{b}\,\underline{c}}\,Y_{\underline{a}\,\underline{b}\,\underline{c}} + \gamma'\ \mathrm{tr}(\mathcal{R}^{\underline{a}\,\underline{b}}\mathcal{R}_{\underline{a}\,\underline{b}})\Bigg]\,, \tag{4.6}$$

and in this expression L is replaced by the expression in (4.4). When this is done two points are made obvious. Firstly, this action is not in the string-frame formulation. This follows in particular since the object $L_{\underline{abc}}$ depends on the dilaton through (4.4). From the superspace point of view this was already obvious due to the field dependence exhibited by (4.1). A string-frame formulation of the dual theory does exist after additional field redefinitions are applied to (4.5) and (4.6).

Secondarily, the Chern-Simons term does not actually appear in this action. One can perform an integration-by-part on the first term on the second line of (4.6) and this leads to a term

$$L^{\underline{a}\,\underline{b}\,\underline{c}}\,Q_{\underline{a}\,\underline{b}\,\underline{c}}\;\propto\;\epsilon^{\underline{a}_1\cdots\underline{a}_6\,\underline{b}_1\,\underline{b}_2\,\underline{c}_1\,\underline{c}_2}\;M_{\underline{a}_1\cdots\underline{a}_6}\;\mathrm{tr}(\mathcal{R}_{\underline{b}_1\,\underline{b}_2}\mathcal{R}_{\underline{c}_1\,\underline{c}_2})\,, \tag{4.7}$$

which can be seen to be precisely the term required by the dual Green-Schwarz mechanism for anomaly cancellation first given in [21]. Notice the change of sign of the L-squared term in (4.5) and (4.6) compared to (3.7) and (3.8). This is the usual sign-flip seen between theories connected by Poincaré duality.

Indeed, now even the variation with respect to the dilaton becomes complicated since $L_{\underline{abc}}$ appears in the connection. However, just marking the variation and using $\delta L = -4L\delta\Phi$ only in the most obvious terms, one ends again with the combination of variations $f(\mathcal{E})$ near the terms of the equation for the dilaton in the theory with two-form (3.13),

$$\delta_\Phi\mathcal{L}_d \sim -4\,e^{-1}\,e^{4\Phi}\;[\;\mathcal{E}_g + 2\mathcal{E}_\Phi\;]\;\delta\Phi + \gamma' f(\mathcal{E})\,. \tag{4.8}$$

The variation with respect to the six-form M is computed in the same manner. As a "miracle" the combination $f(\mathcal{E})$ surprisingly appears again and one simply obtains,

$$\delta_M\mathcal{L}_d\;\sim\;-\frac{2}{3}\frac{1}{4!6!}\epsilon^{\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}\,[6]}\hat{\mathcal{E}}_{\widetilde{B}_{\underline{abcd}}}\delta M_{[6]} + \gamma' f(\mathcal{E})\,. \tag{4.9}$$

So the final conclusion is that in the dual theory, the component action in (4.6) is compatible with the equations of motion derived from superspace for the dual theory.

5. Comparison with a component level investigation

Next, let us study the relationship of the lagrangian (3.8) with the component lagrangian in [23]. A quick look to the component lagrangian in [23] convinces us that using just rescalings of the fields it can be written in the form

$$\begin{aligned}\hat{\mathcal{L}} = \mathrm{e}^{-1}\,e^{4\Phi}\Big[&\mathcal{R}(\omega) + 16\,(\mathrm{e}^{\underline{a}}\Phi)\,(\mathrm{e}_{\underline{a}}\Phi) - \frac{1}{3}\,G^{\underline{a}\,\underline{b}\,\underline{c}}\,(\,G_{\underline{a}\,\underline{b}\,\underline{c}} + 2\gamma'\,Q_{\underline{a}\,\underline{b}\,\underline{c}}\,) + \\ &+\gamma'\mathrm{tr}(\widehat{\mathcal{R}}^{\underline{a}\,\underline{b}}\,\widehat{\mathcal{R}}_{\underline{a}\,\underline{b}})\Big]\,,\end{aligned} \tag{5.1}$$

where hatted objects are defined using a Lorentz connection $\widehat{\Omega}$, which may differ from ours by its torsion. In order to compare this to our lagrangian (3.8), let us write the difference as

$$\mathrm{e}^{-1}\,e^{-4\Phi}\Big(\,\mathcal{L}-\hat{\mathcal{L}}\Big) = -\frac{2}{3}\gamma' G^{\underline{a}\,\underline{b}\,\underline{c}}\,\Big(Q-\hat{Q}\Big)_{\underline{a}\,\underline{b}\,\underline{c}} - \frac{2}{3}\gamma' G^{\underline{a}\,\underline{b}\,\underline{c}}\,Y_{\underline{a}\,\underline{b}\,\underline{c}} + \gamma'\;\mathrm{tr}\,\Big(\,\mathcal{R}_{\underline{a}\,\underline{b}}\,\mathcal{R}^{\underline{a}\,\underline{b}} - \widehat{\mathcal{R}}_{\underline{a}\,\underline{b}}\,\widehat{\mathcal{R}}^{\underline{a}\,\underline{b}}\Big)\,. \tag{5.2}$$

Observe that the difference is in fact a GY term. The question is whether this additional term can be removed by field redefinitions.

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First of all, notice, that only redefinitions at zero order of the Lorentz connection can affect this difference at first order. For example, let us redistribute the torsion in the connection using a real parameter k in the simplest way,

$$\Omega_{\underline{a}\,\underline{b}\,\underline{c}} = \omega_{\underline{a}\,\underline{b}\,\underline{c}} - L_{\underline{a}\,\underline{b}\,\underline{c}} = \widehat{\Omega}_{\underline{a}\,\underline{b}\,\underline{c}} + \chi_{\underline{a}\,\underline{b}\,\underline{c}}\,, \tag{5.3}$$

$$\widehat{\Omega}_{\underline{a}\,\underline{b}\,\underline{c}} = \omega_{\underline{a}\,\underline{b}\,\underline{c}} - (1-k)L_{\underline{a}\,\underline{b}\,\underline{c}}\,, \tag{5.4}$$

$$\chi_{\underline{a}\,\underline{b}\,\underline{c}} = -kL_{\underline{a}\,\underline{b}\,\underline{c}}\,. \tag{5.5}$$

This can be seen as a shift in the connection of type (A.13), which is frequently used to find conventional constraints in supergravity. For $k=0$ in fact there is "no redefinition", for $k=1$ the new connection $\widehat{\Omega}=\omega$ is torsionfree, while for $k=2$ the sign of the torsion flips.

How does this shift in the connection affect the form of the lagrangian? One computes the changes in the Chern-Simons term and the curvature squared term using (A.16) and respectively (A.15)

$$\begin{aligned}\frac{2}{3}G^{\underline{a}\,\underline{b}\,\underline{c}}\left(Q-\widehat{Q}\right)_{\underline{a}\,\underline{b}\,\underline{c}} \sim{}& -4k\left[\mathcal{R}_{\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}} + 2k\left(1-\frac{k}{3}\right)G_{\underline{a}\,\underline{c}}{}^{\underline{k}}\,G_{\underline{b}\,\underline{d}\,\underline{k}}\right]G^{\underline{a}\,\underline{b}\,\underline{l}}G^{\underline{c}\,\underline{d}}{}_{\underline{l}} - \\ & -4k\,e^{-4\Phi}\hat{\mathcal{E}}_{B_{\underline{a}\underline{b}}}\,\widehat{\Omega}_{\underline{b}\,\underline{e}\,\underline{k}}\,G_{\underline{a}}{}^{\underline{e}\,\underline{k}} + \mathcal{O}(\gamma')\,,\end{aligned} \tag{5.6}$$

$$-\frac{2}{3}G^{\underline{a}\,\underline{b}\,\underline{c}}Y_{\underline{a}\,\underline{b}\,\underline{c}} = 8\left[\mathcal{R}_{\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}} + \frac{4}{3}G_{\underline{a}\,\underline{c}}{}^{\underline{k}}\,G_{\underline{b}\,\underline{d}\,\underline{k}}\right]G^{\underline{a}\,\underline{b}\,\underline{l}}\,G^{\underline{c}\,\underline{d}}{}_{\underline{l}}\,, \tag{5.7}$$

$$\begin{aligned}\mathrm{tr}\left(\mathcal{R}_{\underline{a}\,\underline{b}}\mathcal{R}^{\underline{a}\,\underline{b}} - \widehat{\mathcal{R}}_{\underline{a}\,\underline{b}}\widehat{\mathcal{R}}^{\underline{a}\,\underline{b}}\right) \sim{}& 2k^2\,(k-2)^2\left[\left(G_{\underline{a}\,\underline{b}}{}^{\underline{k}}\,G_{\underline{c}\,\underline{d}\,\underline{k}} - G_{\underline{a}\,\underline{c}}{}^{\underline{k}}\,G_{\underline{b}\,\underline{d}\,\underline{k}}\right)\right]G^{\underline{a}\,\underline{b}\,\underline{l}}\,G^{\underline{c}\,\underline{d}}{}_{\underline{l}} - \\ & -4k(k-2)\left[\hat{\mathcal{E}}_{\eta_{\underline{k}\underline{l}}}G_{\underline{l}}{}^{\underline{c}\,\underline{d}} + \nabla^{\underline{k}}(\,e^{-4\Phi}\hat{\mathcal{E}}_{B_{\underline{c}\,\underline{d}}})\right]G_{\underline{k}\,\underline{c}\,\underline{d}} + \\ & +2k(k-2)\,\mathcal{R}_{\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}}\,G^{\underline{a}\,\underline{b}\,\underline{k}}\,G^{\underline{c}\,\underline{d}}{}_{\underline{k}} + \mathcal{O}(\gamma')\,,\end{aligned} \tag{5.8}$$

and finally we find

$$\begin{aligned}e^{-4\Phi}\left(\mathcal{L}-\widehat{\mathcal{L}}\right) \sim{}& 2(k-2)^2\gamma'\mathcal{R}_{\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}}\,G^{\underline{a}\,\underline{b}\,\underline{k}}\,G^{\underline{c}\,\underline{d}}{}_{\underline{k}} + \\ & +(k-2)^2\gamma'\left[2k^2G_{\underline{a}\,\underline{b}}{}^{\underline{k}}\,G_{\underline{c}\,\underline{d}\,\underline{k}} + \frac{2}{3}(k+4)G_{\underline{a}\,\underline{c}}{}^{\underline{k}}\,G_{\underline{b}\,\underline{d}\,\underline{k}}\right]G^{\underline{a}\,\underline{b}\,\underline{l}}\,G^{\underline{c}\,\underline{d}}{}_{\underline{l}} - \\ & -4k(k-2)\gamma'\left[\widehat{\mathcal{E}}_{\eta_{\underline{c}\underline{l}}}G_{\underline{l}}{}^{\underline{k}\,\underline{d}} + \nabla^{\underline{k}}(\,e^{-4\Phi}\hat{\mathcal{E}}_{B_{\underline{c}\,\underline{d}}})\right]G_{\underline{k}\,\underline{c}\,\underline{d}} - \\ & -4k\gamma'\,e^{-4\Phi}\,\widehat{\mathcal{E}}_{B_{\underline{c}\,\underline{d}}}\,\widehat{\Omega}_{\underline{c}\,\underline{e}\,\underline{f}}\,G_{\underline{d}}{}^{\underline{e}\,\underline{f}}\,.\end{aligned} \tag{5.9}$$

Observe, that for $k=0$, indeed, the difference is equal to the GY term, while for $k=2$, the difference is a term proportional to the equation of motion for the antisymmetric tensor at zero order:

$$\mathcal{L}-\widehat{\mathcal{L}} \sim -8\gamma'\,\widehat{\mathcal{E}}_{B_{\underline{c}\,\underline{d}}}\,\widehat{\Omega}_{\underline{c}\,\underline{e}\,\underline{f}}\,G_{\underline{d}}{}^{\underline{e}\,\underline{f}}\,. \tag{5.10}$$

At first sight it seems that the change of sign of the torsion in the Lorentz connection just exchanges the GY term to another "unwanted" one. However, correction terms which are proportional to equations of motion can be absorbed by field redefinitions involving the perturbation parameter and therefore $\mathcal{L}$ and $\hat{\mathcal{L}}$ are equivalent.

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Indeed, let us consider the expression

$$S[\phi] + \gamma' \int dx^n \frac{\delta S}{\delta \phi} \mathcal{F}(\phi) \,, \tag{5.11}$$

with $S[\phi]$ an action for the fields ϕ, $\frac{\delta S}{\delta \phi} = 0$ the equations of motion for the fields ϕ, $\mathcal{F}(\phi)$ an arbitrary function of the fields ϕ and γ' an infinitesimal parameter. Now consider the field redefinitions

$$\phi' = \phi + \gamma' \mathcal{F}(\phi) \,, \tag{5.12}$$

and develop $S[\phi']$ around ϕ using that γ' is infinitesimal. Then one obtains

$$S[\phi'] = S[\phi] + \gamma' \int dx^n \frac{\delta S}{\delta \phi} \mathcal{F}(\phi) + \mathcal{O}(\gamma'^2) \,. \tag{5.13}$$

As a conclusion, we have proven here that the bosonic lagrangian (3.8), based on the superspace geometry proposed in [5] is equivalent to the component-level first-order corrected supergravity lagrangian of [23].

6. Conclusion

With this present work, we have re-engaged in a discussion that began almost twenty years ago. We hope that this has presented in the clearest possible terms the proposal given in [5]. In particular, we gave the bosonic lagrangian corresponding to the superspace geometry proposed in [5] and we showed that this lagrangian is equivalent to the gravity part (the YM coupling constant is set to zero) of the first-order corrected anomaly-free supergravity lagrangian. The issue of duality in the framework of superspace geometry is discussed and the dual theory is also presented.

Aside from issues connected with the controversy over the form of the lowest order terms (deformations) in the slope-parameter expansion of the supergeometry, the technique in this paper of using superspace to generate equations of motion, taking the limit of vanishing fermions and then integrating the resulting bosonic equations to derive an action insures that the bosonic limit reached in this manner is consistent with supersymmetry. We believe this paper marks one of the first times this set of steps has been applied to a supergravity theory.

A discussion of these results and their relation to some of the "conventional wisdom" on this topic (based on [8]) will be treated in a separate publication [9].

> *"It's no exaggeration to say the undecideds could go one way or another."* - George H. W. Bush

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A. Definitions & conventions

The basic tool we use is ten dimensional chiral superspace with structure group SO(1,9). Definitions and properties (such as multiplication table and Fierz identities) of ten dimensional chiral sigma matrices we adopted here can be found in [17]. Given the super frame $E^{\mathcal{A}} = (E^{\underline{a}},\ E^{\alpha})$, conventions for superforms and Leibniz rule for the exterior derivative are

$$\omega = \frac{1}{p!} E^{\mathcal{A}_1} \ldots E^{\mathcal{A}_p}\, \omega_{\mathcal{A}_p \ldots \mathcal{A}_1}\,, \tag{A.1}$$

$$\mathrm{d}(\omega_p \omega_q) = \omega_p (\mathrm{d}\omega_q) + (-)^q (\mathrm{d}\omega_p)\omega_q\,. \tag{A.2}$$

Representation matrices acting on the tangent space are blockdiagonal,

$$X = \begin{pmatrix} X_b{}^a & 0 \\ 0 & X_\beta{}^\alpha \end{pmatrix}, \tag{A.3}$$

and the vectorial and spinorial representations are related by the two-index sigma matrix,

$$X_\alpha{}^\beta = \frac{1}{4}(\sigma^{\underline{a}\,\underline{b}})_\alpha{}^\beta X_{\underline{a}\,\underline{b}}\,, \qquad X_{\underline{a}\,\underline{b}} = -\frac{1}{8}(\sigma_{\underline{a}\,\underline{b}})_\alpha{}^\beta X_\beta{}^\alpha\,. \tag{A.4}$$

As soon as the action of the structure group is fixed,

$$\delta E = \beta E X\,, \tag{A.5}$$

the covariant derivative

$$\nabla E = \mathrm{d}E + \alpha E \Omega \tag{A.6}$$

can be defined using the Lorentz connection Ω with transformation law

$$\delta\Omega = -\beta\,(\mathrm{d}X + \alpha X \cdot \Omega)\,. \tag{A.7}$$

The torsion T, the curvature $\mathcal{R}$ and field strengths F_p of an abelian $(p-1)$-form are defined by

$$\nabla E = \gamma T, \qquad \mathcal{R} = \mathrm{d}\Omega + \alpha\Omega\Omega, \qquad F_p = \mathrm{d}A_{p-1}\,, \tag{A.8}$$

and they satisfy the following Bianchi identities

$$\gamma\nabla T = \alpha E\mathcal{R}\,, \qquad \nabla\mathcal{R} = 0, \qquad \mathrm{d}F_p = 0\,. \tag{A.9}$$

The curvature in particular appears in the double covariant derivative of covariant vectors

$$\nabla\nabla u = \alpha u \mathcal{R}\,. \tag{A.10}$$

Dragon's theorem states that in supergravity the Bianchi identity for the torsion together with (A.10) implies that the Bianchi identity for the curvature is automatically satisfied.

The Chern-Simons form

$$Q = \mathrm{tr}\left(\Omega\mathcal{R} - \frac{\alpha}{3}\Omega\Omega\Omega\right) \tag{A.11}$$

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satisfies

$$\mathrm{d}Q = \mathrm{tr}(\mathcal{R}\mathcal{R})\,. \tag{A.12}$$

Finally, let us consider a redefinition

$$\Omega = \hat{\Omega} + \chi \tag{A.13}$$

of the connection. This shift in the connection affects the torsion, the curvature and the Chern-Simons form in the following way:

$$\begin{aligned}
\gamma(T-\hat{T}) &= \alpha E\chi\,, && \text{(A.14)}\\
\mathcal{R}-\hat{\mathcal{R}} &= \nabla\chi - \alpha\chi\chi\,, && \text{(A.15)}\\
Q-\hat{Q} &= \mathrm{tr}\left(2\mathcal{R}\chi - \chi\nabla\chi + \frac{2\alpha}{3}\chi\chi\chi + \mathrm{d}(\Omega\chi)\right)\,. && \text{(A.16)}
\end{aligned}$$

Let us display the above relations in terms of form-components. First of all, (A.10) gives the algebra of covariant derivatives acting on covariant vectors

$$(\nabla_P, \nabla_{\mathcal{B}})\, u^{\mathcal{A}} = -\gamma T_{P\mathcal{B}}{}^{\mathcal{F}}\nabla_{\mathcal{F}}u^{\mathcal{A}} + \alpha\mathcal{R}_{P\mathcal{B}\mathcal{F}}{}^{\mathcal{A}}u^{\mathcal{F}}\,. \tag{A.17}$$

The Bianchi identities become

$$\begin{aligned}
\gamma\nabla_{(\nabla}T_{P\mathcal{B})}{}^{\mathcal{A}} + \gamma^2 T_{(\nabla P|}{}^{\mathcal{F}}T_{\mathcal{F}|\mathcal{B})}{}^{\mathcal{A}} - \alpha\mathcal{R}_{(\nabla P\mathcal{B})}{}^{\mathcal{A}} &= 0 && \text{(A.18)}\\
\nabla_{(\mathcal{A}_1}F_{\mathcal{A}_2\ldots\mathcal{A}_{p+1})} + \gamma\frac{p}{2}T_{(\mathcal{A}_1\mathcal{A}_2|}{}^{\mathcal{F}}F_{\mathcal{F}|\mathcal{A}_3\ldots\mathcal{A}_{p+1})} &= 0\,. && \text{(A.19)}
\end{aligned}$$

The components of the Chern-Simons form are

$$Q_{\mathcal{A}\mathcal{B}P} = \mathrm{tr}\left(\frac{1}{2}\Omega_{(\mathcal{A}}\mathcal{R}_{\mathcal{B}P)} + \frac{\alpha}{3}\Omega_{(\mathcal{A}}\Omega_{\mathcal{B}}\Omega_{P)}\right)\,, \tag{A.20}$$

while the redefinitions take the form

$$\begin{aligned}
\gamma(T-\hat{T})_{P\mathcal{B}}{}^{\mathcal{A}} &= \alpha\chi_{(P\mathcal{B})}{}^{\mathcal{A}}\,,\\
(\mathcal{R}-\hat{\mathcal{R}})_{\mathcal{B}\mathcal{A}} &= \nabla_{(\mathcal{B}}\chi_{\mathcal{A})} + \gamma T_{\mathcal{B}\mathcal{A}}{}^{\mathcal{F}}\chi_{\mathcal{F}} + \alpha\chi_{(\mathcal{B}}\chi_{\mathcal{A})}\,,\\
(Q-\hat{Q})_{P\mathcal{B}\mathcal{A}} &= \mathrm{tr}\Big[\mathcal{R}_{(P\mathcal{B}}\chi_{\mathcal{A})} - \chi_{(P}\left(\nabla_{\mathcal{B}}\chi_{\mathcal{A})} + \frac{\gamma}{2}T_{\mathcal{B}\mathcal{A})}{}^{\mathcal{F}}\chi_{\mathcal{F}} + \frac{2\alpha}{3}\chi_{\mathcal{B}}\chi_{\mathcal{A})}\right) -\\
&\qquad - \nabla_{(P}(\Omega_{\mathcal{B}}\chi_{\mathcal{A})} - \frac{\gamma}{2}\Omega_{(\mathcal{F}}\chi_{(\mathcal{A})}T_{P\mathcal{B})}{}^{\mathcal{F}}\Big]\,. && \text{(A.21)}
\end{aligned}$$

The conventions of Wess and Bagger correspond to the choice $\alpha = 1$, $\gamma = 1$, while the conventions in [5] correspond to $\alpha = -1$, $\gamma = -1$. Also, the Chern-Simons term denoted by X in [5] is $X = -Q$.

The graviton and gravitino is identified in the super frame $E^{\mathcal{A}} = (E^{\underline{a}}, E^{\alpha})$,

$$E^{\underline{a}}| = \mathrm{d}x^{\underline{m}}\mathrm{e}_{\underline{m}}{}^{\underline{a}}, \qquad E^{\alpha}| = \frac{1}{2}\mathrm{d}x^{\underline{m}}\psi_{\underline{m}}{}^{\alpha}\,. \tag{A.22}$$

The torsion, $T = -\nabla E$, satisfies the Bianchi identity

$$\nabla T = E\mathcal{R}\,. \tag{A.23}$$

The two-form gauge potential of the pure 10 dimensional supergravity multiplet is identified in a two-form on the superspace

$$B\| = \frac{1}{2} \mathrm{d}x^{\underline{m}} \mathrm{d}x^{\underline{n}} B_{\underline{n}\,\underline{m}}\,. \tag{A.24}$$

Its fieldstrengths $G = \mathrm{d}B$ satisfies the Bianchi identity

$$\mathrm{d}G = 0\,. \tag{A.25}$$

The Green-Schwarz mechanism teaches us that in order to deal with anomaly free supergravity the field strength of the antisymmetric tensor has to be accompanied by both the Yang-Mills and gravitational Chern-Simons terms. Here we consider only the gravitational part.

$$Q = \mathrm{tr}\left(\mathcal{R}\Omega + \frac{1}{3}\Omega\Omega\Omega\right), \qquad \mathrm{d}Q = \mathrm{tr}(\mathcal{R}\mathcal{R})\,. \tag{A.26}$$

Therefore, it is convenient in general to define a new object on superspace,

$$H \doteq G + \gamma' Q\,, \tag{A.27}$$

and consider the Bianchi identity satisfied by this three-form H,

$$\mathrm{d}H = \gamma' \,\mathrm{tr}(\mathcal{R}\mathcal{R})\,. \tag{A.28}$$

The six-form gauge potential of the dual pure 10 dimensional supergravity multiplet is identified in a six-form on the superspace

$$M\| = \frac{1}{6!} \mathrm{d}x^{\underline{m}_1} \ldots \mathrm{d}x^{\underline{m}_6} M_{\underline{m}_6 \cdots \underline{m}_1}\,. \tag{A.29}$$

Its fieldstrengths $N = \mathrm{d}B$ satisfies the Bianchi identity

$$\mathrm{d}N = 0\,. \tag{A.30}$$

B. Variations

For arbitrary variation of the connection $\delta\Omega$ the curvature squared terms and the Chern-Simons form Q change according to

$$\delta \;\mathrm{tr}\left(\mathcal{R}^{\underline{ab}} \mathcal{R}_{\underline{ab}}\right) = -4 \;\mathrm{tr}\left[(\nabla_{\underline{a}} \mathcal{R}^{\underline{ab}})\, \delta\Omega_{\underline{b}}\right] + 4\, \partial_{\underline{m}}\Big(\mathrm{e}_{\underline{a}}{}^{\underline{m}} \;\mathrm{tr}(\mathcal{R}^{\underline{ab}}\, \delta\Omega_{\underline{b}})\Big)\,, \tag{B.1}$$

$$\delta Q = \mathrm{tr}\left[\; 2\mathcal{R}\delta\Omega + \mathrm{d}(\Omega\delta\Omega) \;\right]. \tag{B.2}$$

The scalar curvature transforms also:

$$\delta\mathcal{R} = \mathrm{e}_{\underline{a}}{}^{\underline{m}}\, \mathrm{e}_{\underline{b}}{}^{\underline{n}}\, \delta\mathcal{R}_{\underline{m}\,\underline{n}}{}^{\underline{a}\,\underline{b}} \tag{B.3}$$

$$= 2\mathrm{e}_{\underline{a}}{}^{\underline{m}}\, \partial_{\underline{m}}(\delta\Omega_{\underline{b}}{}^{\underline{a}\,\underline{b}}) - T_{\underline{a}\,\underline{b}}{}^{\underline{c}}\, \delta\Omega_{\underline{c}}{}^{\underline{a}\,\underline{b}}\,. \tag{B.4}$$

In the case where $\delta\Omega_{\underline{a}\,\underline{b}\,\underline{c}} = \frac{1}{2}\delta T_{\underline{a}\,\underline{b}\,\underline{c}}$ with totally antisymmetric torsion this yields $\delta\mathcal{R} = -\delta(\frac{1}{4} T_{\underline{a}\,\underline{b}\,\underline{c}} T^{\underline{a}\,\underline{b}\,\underline{c}})$. In particular this implies also that the combination $\mathcal{R} + \frac{1}{4} T_{\underline{a}\,\underline{b}\,\underline{c}} T^{\underline{a}\,\underline{b}\,\underline{c}}$ is independent of a redefinition (A.13) provided that χ is totally antisymmetric.

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Using the above formulae one may compute the following variations with respect to an object $L_{\underline{abc}}$ appearing in the Lorentz connection as

$$\Omega_{\underline{a}\,\underline{b}\,\underline{c}} = \omega_{\underline{a}\,\underline{b}\,\underline{c}} - L_{\underline{a}\,\underline{b}\,\underline{c}}\,, \tag{B.5}$$

with ω the torsion free spin connection:

$$\begin{aligned}
\mathrm{e}^{-1}\, e^{4\Phi}\delta_L\,\mathrm{tr}\left(\mathcal{R}^{\underline{a}\,\underline{b}}\mathcal{R}_{\underline{a}\,\underline{b}}\right) &\sim -4\mathrm{e}^{-1}\nabla^{\underline{a}}(\,e^{4\Phi}\mathcal{R}_{\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}})\,\delta L^{\underline{b}\,\underline{c}\,\underline{d}} + \\
&\quad + 4\mathrm{e}^{-1}\,e^{4\Phi}\mathcal{R}_{\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}}\,L^{\underline{a}\,\underline{b}}{}_{\underline{k}}\,\delta L^{\underline{k}\,\underline{c}\,\underline{d}} \\
&\quad + \mathcal{O}(\gamma')\,, && \text{(B.6)}\\
-\frac{2}{3}\mathrm{e}^{-1}\,e^{4\Phi}L^{\underline{a}\,\underline{b}\,\underline{c}}\delta_L Q_{\underline{a}\,\underline{b}\,\underline{c}} &\sim 4\mathcal{E}_{B_{\underline{kl}}}\Omega_{\underline{k}}{}^{\underline{a}\,\underline{b}}\,\delta L_{\underline{l}\,\underline{a}\,\underline{b}} - 4\mathrm{e}^{-1}\,e^{4\Phi}\,\mathcal{R}_{\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}}\,L^{\underline{a}\,\underline{b}}{}_{\underline{k}}\delta L^{\underline{k}\,\underline{c}\,\underline{d}} + \\
&\quad + \mathcal{O}(\gamma') - && \text{(B.7)}\\
-\frac{2}{3}\mathrm{e}^{-1}\,e^{4\Phi}L^{\underline{a}\,\underline{b}\,\underline{c}}\delta_L Y_{\underline{a}\,\underline{b}\,\underline{c}} &= 4\,e^{4\Phi}(\mathcal{R}_{\underline{a}\,\underline{bc}\,\underline{d}} + \mathcal{R}_{\underline{c}\,\underline{da}\,\underline{b}})L_{\underline{k}}{}^{\underline{a}\,\underline{b}}\,\delta L^{\underline{k}\,\underline{c}\,\underline{d}} - \\
&\quad - \frac{2}{3}\,e^{4\Phi}L_{\underline{a}\,\underline{b}}{}^{\underline{k}}\,L_{\underline{c}\,\underline{d}\,\underline{k}}\delta_L\left(\mathcal{E}_{\widetilde{B}_{\underline{abcd}}}\right) + \mathcal{O}(\gamma')\,. && \text{(B.8)}
\end{aligned}$$

However, the first term in the variation (B.6) may be recast in the form

$$\begin{aligned}
-4\left[\,\nabla^{\underline{a}}(\,e^{4\Phi}\mathcal{R}_{\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}})\right]\delta L^{\underline{b}\,\underline{c}\,\underline{d}} &\sim -4\,e^{4\Phi}(\mathcal{R}_{\underline{a}\,\underline{bc}\,\underline{d}} + \mathcal{R}_{\underline{c}\,\underline{da}\,\underline{b}})L_{\underline{k}}{}^{\underline{a}\,\underline{b}}\,\delta L^{\underline{k}\,\underline{c}\,\underline{d}} + \\
&\quad + 8\left[e^{4\Phi}\nabla^{\underline{a}}\left(e^{-4\Phi}\hat{\mathcal{E}}_{B_{\underline{b}\,\underline{c}}}\right)\right]\,\delta L_{\underline{ab}\,\underline{c}} + \\
&\quad + 8\left[\left(e^{4\Phi}\hat{\mathcal{E}}_{\eta_{\underline{ak}}} - \hat{\mathcal{E}}_{B_{\underline{ak}}}\right)L_{\underline{k}}{}^{\underline{b}\,\underline{c}}\right]\delta L_{\underline{ab}\,\underline{c}} + \\
&\quad + \frac{2}{3}\,e^{4\Phi}\left[\,L_{\underline{a}\,\underline{b}}{}^{\underline{k}}\,L_{\underline{c}\,\underline{d}\,\underline{k}} - \frac{1}{4!}\mathcal{E}_{\widetilde{B}\underline{abcd}}\,\right]\,\delta_L\left(\mathcal{E}_{\widetilde{B}_{\underline{abcd}}}\right) + \\
&\quad + \mathcal{O}(\gamma')\,. && \text{(B.9)}
\end{aligned}$$

Now observe, that the sum of the variations written above is expressed as a combination of the equations we derived from superspace geometry. We denote this combination of variations simbolically by $f(\mathcal{E})$:

$$f(\mathcal{E}) \doteq \mathrm{e}^{-1}\,e^{4\Phi}\delta_L\,\mathrm{tr}\left(\mathcal{R}^{\underline{a}\,\underline{b}}\mathcal{R}_{\underline{a}\,\underline{b}}\right) - \frac{2}{3}\mathrm{e}^{-1}\,e^{4\Phi}\,L^{\underline{a}\,\underline{b}\,\underline{c}}\,\delta_L\,(\;Q + Y\;)_{\underline{a}\,\underline{b}\,\underline{c}}\,, \tag{B.10}$$

$$\begin{aligned}
f(\mathcal{E}) &\sim 4\mathcal{E}_{B_{\underline{kl}}}\Omega_{\underline{k}}{}^{\underline{a}\,\underline{b}}\delta L_{\underline{l}\,\underline{a}\,\underline{b}} + \\
&\quad + 8\left[e^{4\Phi}\nabla^{\underline{a}}\left(e^{-4\Phi}\hat{\mathcal{E}}_{B_{\underline{bc}}}\right) + \left(e^{4\Phi}\hat{\mathcal{E}}_{\eta_{\underline{a}\,\underline{k}}} - \hat{\mathcal{E}}_{B_{\underline{a}\,\underline{k}}}\right)\,L_{\underline{k}}{}^{\underline{b}\,\underline{c}}\right]\,\delta L_{\underline{a}\,\underline{b}\,\underline{c}} - \\
&\quad - \frac{2}{3}\,e^{4\Phi}\frac{1}{4!}\mathcal{E}_{\widetilde{B}\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}}\delta_L\left(\mathcal{E}_{\widetilde{B}_{\underline{abcd}}}\right) + \\
&\quad + \mathcal{O}(\gamma'). && \text{(B.11)}
\end{aligned}$$

Therefore the superspace equations imply the vanishing of the above combination for an arbitrary variation of the object $L_{\underline{abc}}$. In particular, this is valid at zero order in γ' both for the anomaly free supergravity and for its dual.

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